THREE PAPERS ON STRATEGIC DECISION MAKING IN DIGITALLY MEDIATED MARKETS
(Chapter 2)

by

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Bundling Information Goods: A Study of Competing Firms Facing Heterogeneous Consumers

1 Introduction

Information goods such as books, news stories, scholarly articles, music recordings, movies, computer games, and software are characterized by high fixed (first-copy) costs, but low costs for the production of additional copies. This cost structure is greatly exaggerated for digital information goods. King and Tenopir (2005) observe that the maturation and integration of communication technologies and the economics of the journal system, particularly pricing of traditional journal subscriptions and access to digital full-text databases through site licensing and packages have the potential either of destroying the scholarly journal system or substantially enhancing its considerable usefulness and value. They argue that "the new technologies should, if deployed with care, enhance the journal system (. . .), but contemporary pricing policies have been a greater threat to the journal system."

One of the challenges for information-good producers is that linear marginal-cost pricing for electronic information goods cannot result in efficient production or distribution because near-zero prices would not recover the initial fixed costs (Varian, 1995). On the other hand, a price above the marginal cost creates a deadweight loss and therefore is inefficient. However, the flexibility of digital technology permits a wide range of responses to the cost problem. For example, by selling individual items as well as bundles such as journals, CDs, and software packages, firms can potentially attract more consumers than with either pricing scheme.[1] Offering both schemes is much more feasible for digital goods available online than for information goods produced and distributed on physical media.

In this chapter I study how product configuration flexibility introduced by digital technology may affect competing producers of information goods and consumers. An important contribution of this work, for the analysis of both monopoly and competitive bundling, is that I study consumers with heterogeneous preferences.

[1] This is assuming consumers' tastes are heterogeneous enough.

Several authors have provided valuable insights about bundling under a simplifying assumption that consumers are characterized by a single variable and are ex ante homogeneous (e.g., Schmalensee, 1984; Zahray & Sirbu, 1990; Bakos & Brynjolfsson, 1999). However, the assumption of such preferences is quite restrictive: for example, it implies that a monopolist will maximize social welfare but will extract all available consumers' surplus (Bakos & Brynjolfsson, 1999). In this chapter, I study environments with high consumer diversity. Following Chuang and Sirbu (1998), I assume two-dimensional consumer preferences, which capture not only the consumers' reservation price for a set of goods, but also correlations between individual valuations within the set. The latter is particularly important in the context of mixed bundling.

For tractability, I restrict my attention to three types of product configurations, or bundling strategies: selling items individually (at the same per-item price), pure bundling (the whole collection of items is sold together), and mixed bundling (consumers are offered a choice between the two previous options). With N goods, there are $2^N - 1$ different bundles that could be offered, each at a potentially unique price, and consumer preferences would need to be specified on this $(2^N - 1)$-dimensional space. The profit-maximization problem for setting prices is NP-hard and generally considered to be computationally intractable for modern computers when N is only moderately large.[2] In this study, the collection size of a single firm varies from 50 to 150. Complex pricing schemes would also create intractable consumer information-processing problems and impractical transactions costs.

There are several reasons why a firm may choose to bundle its goods. Previous authors provide at least five reasons for selling two or more products in a single package:

1. Cost savings in production and transactions associated with package selling in presence of economies of scale (e.g., Coase, 1960; Bakos & Brynjolfsson, 2000; Demsetz, 1968; Chuang & Sirbu, 1998);

2. Strategic bundling such as *tying* for leveraging market power (e.g., Carbajo, Meza, & Seidmann, 1990; Whinston, 1990);

3. Complementarity (superadditivity) in consumption of bundling components (e.g., Adams & Yellen, 1976; Bakos & Brynjolfsson, 1999; Economides & Viard, 2004);

4. Second-degree price discrimination when marginal valuations are additive or subadditive (e.g., Adams & Yellen, 1976; McAfee, McMillan, & Whinston, 1989; Varian, 2000, 1989; Bakos & Brynjolfsson, 1999; Chuang & Sirbu, 1998); and

[2] My empirical analysis also shows that a monopoly does not benefit from splitting its collections into two and applying independent bundling schemes to them. This holds for all of the preference distributions I have analyzed. This part of the analysis is not reported here, but is available upon request.

5. Reducing buyer diversity (via aggregation) (e.g., Schmalensee, 1984; Varian, 1989; Salinger, 1995; Bakos & Brynjolfsson, 1999, 2000).

The first point concerns savings on the supply side, whereas the last three represent bundling as a tool for extracting consumer surplus. The literature under the second category studies how a firm with monopoly power in one market can use its leverage provided by this power to forclose sales in, and thereby monopolize, a second market. Producers of information goods may choose to bundle for any of these reasons. In the paper-based production of scholarly articles, for example, packaging articles into journals and journals into subscriptions saves non-negligible reproduction and distribution costs. Software is often pre-installed on personal computers, which leverages the market power of the software producer. Software packages typically contain complementary programs. Producers of business news sub-bundle general and time-sensitive information – such as stock quotations or financial news – and delay the latter for those who are not willing to pay a premium. This is an example of price discrimination: the scheme induces casual readers and readers who use the information for business purposes to self-select into appropriate consumption groups. The aggregation effect often manifests itself in bundles of music recordings (CDs), articles (journals or magazines), TV shows (TV channels), etc. The consumers' willingness to pay for such bundles varies less than their willingness to pay for individual components. In this chapter, however, I am concerned only with two of all possible reasons for bundling: price discrimination and reducing buyer diversity. Also, I study the use of bundling schemes when firms *compete* in the selling of information goods. Thus, I address the interaction between market structure and bundled pricing strategies.[3]

In particular, I consider two firms, each producing differentiated collections of information goods. I assume that they have zero reproduction and distribution costs and face a population of consumers with heterogeneous tastes. I model heterogeneous tastes as a valuation function that depends on two parameters. One parameter is the consumer value of her most favored item on the market, and the other is the percentage of the items on the market that she values positively. My preference model is a generalization of Chuang and Sirbu's monopoly model (Chuang & Sirbu, 1998) to two firms producing differentiated collections of items.[4] The firms can choose among three bundling schemes: they can offer individual items, each at the same price (pure unbundling), sell their collection as a single bundle (pure bundling), or offer both options at the same time (mixed bundling).

[3]I restrict my attention to competition in the market for a single differentiated information good. Analysis of bundling on multiple linked markets for strategic purposes, which falls under the second category, is outside of the scope of this chapter.

[4]My production model, however, is a special case of the Chuang-Sirbu model: Chuang and Sirbu assume a positive marginal cost and different levels of economies of scale, while I consider the extreme case of zero marginal cost and, consequently, no economies of scale.

Within this framework, I address several questions. First, can simple mixed bundling effectively sort consumers under competition? How do the equilibrium profits compare to those when the firms are restricted to pure bundling or pure unbundling? How do the pure forms compare to each other? Under monopoly, mixed bundling must perform at least as well as either pure unbundling or pure bundling since it contains both pure strategies as sub-cases: maximizing over a larger set of options may not result in a lower profit. When two firms compete, however, the equilibrium payoffs in a game in which both firms have unrestricted strategy sets can be lower than the equilibrium payoffs in a game in which at least one firm's strategy set is restricted to a smaller set of options. In a price-competition game like the one studied here, a restricted set of pricing schemes may alleviate pricing wars and therefore result in higher profits for the firms.

In cases in which mixed bundling is more profitable in equilibrium than the pure schemes, it is important to know by how much. Pure bundling and pure unbundling are appealing due to their simplicity, which may be an important advantage in practical implementation. My second goal is therefore to quantify the gains from choosing the more complex pricing scheme of mixed bundling.

My third question is how much does competition curb the ability of bundling firms (in pure or mixed forms) to extract value from consumers? Although digital information goods have negligible marginal costs of reproduction and distribution, there may be significant first-copy costs. If competition significantly reduces operating profits, investment in new information products may be low.

Finally, what is the effect of information-good bundling under competition on social welfare, taking into account both consumers' surplus and profits? How do mixed-bundling equilibrium profits, social welfare, and consumer surplus compare to those when the firms are restricted to one of the pure schemes?

Due to the complexity induced by heterogeneous, multi-dimensional consumer preferences over multiple goods sold in various configurations, I have not been able to solve analytically for equilibrium of the pricing game. As in Chapter **??**, I make use of the *empirical game-theoretic methodology* to analyze competitive bundling in a number of specific *environments*.

I find that depending on the consumer preference distribution, any of the three schemes can result in the highest equilibrium profits when two firms compete. While under monopoly mixed bundling is always weakly better due to its ability to sort consumers into different consumption groups, under duopoly it also leads to more aggressive price competition. However, mixed bundling is generally the most profitable scheme, and in the cases when a pure form yields higher profits in equilibrium, the difference from the mixed-bundling equilibrium profits is small. Under monopoly, a pure pricing scheme gives 80–100% of the mixed-bundling profits; under duopoly, at least 75–105%, depending on the preference distribution and the scheme employed by the other firm.

I also find that under competition, the profits of firms that bundle – in pure or mixed form – are lower by up to 21% relative to a monopoly employing the same (combination of) pricing schemes as the duopoly. The market efficiency is up to 16% higher, and the distribution of social welfare shifts toward consumers by up to 22%. These estimates are based on three different distributions of consumer preferences, each analyzed for a monopoly, a symmetric duopoly and a non-symmetric duopoly.

2 Related Work

In their seminal paper on bundling, Adams and Yellen (1976) consider a monopolist producing two goods. They analyze three pricing strategies: component selling (each good priced and sold individually), bundling (both goods sold together), and mixed bundling (consumers offered a choice between buying a bundle or individual components). They show by example that bundling can result in higher revenue than component selling and argue that the profitability of a bundle of different commodities can "stem from its ability to sort customers into groups with different reservation price characteristics, and hence to extract consumer surplus." However, bundling might result in inefficient consumption of components for which the marginal cost of producing that component exceeds its value to a consumer. In general, the profitability of bundling strategies depends on the prevailing level of marginal costs and on the distribution of customer valuations.

McAfee et al. (1989) extend the analysis to general demand functions, and provide conditions under which mixed bundling strictly dominates either of the two pure strategies. Salinger (1995) shows that if valuations for the two goods are not perfectly correlated, the demand for the bundle will be more price-elastic than the sum of the individual demands for each component. As a result, bundling is a way to smooth out idiosyncratic preferences enabling a monopolist to extract more consumers' surplus than is possible through pure components pricing.

Some more recent papers have extended the analysis to more than two goods. Hanson and Martin (1990) tackle the full 2^N bundle pricing problem with N=21. MacKie-Mason and Riveros (1998), and Hitt and Chen (2005) introduce a new alternative: generalized subscriptions. Consumers prepay for G tokens, then select G items from the entire collection after the items are created. In effect, individual-specific sub-bundles are created. Riveros (1999) comparatively evaluates generalized subscriptions against the three most studied strategies.

Brooks, Fay, Das, MacKie-Mason, Kephart, and Durfee (1999), Kephart, Das, Brooks, Durfee, Gazzale, and MacKie-Mason (2001), and Brooks, Gazzale, Das, Kephart, MacKie-Mason, and Durfee (2002) employ machine-learning techniques to explore the space of pricing policies. They are concerned with the trade-off between exploitation and exploration of pricing policies in static and dynamic environments. They show that simple pricing policies with few parameters to learn are more robust in both environment types. Kephart, Das, and MacKie-Mason (2000) acknowledge that information is an experience good. They allow consumers to learn their valuations and the seller to learn pricing parameters simultaneously. They find that dynamic market interactions when there is substantial uncertainty can lead to pathological outcomes if agents are designed with reasonable but not sufficiently adaptive strategies.

Bakos and Brynjolfsson (1998) illustrate that bundling with N goods is strictly preferred to component pricing when marginal cost is zero, consumer item preferences are homogeneous and identically distributed, and N is sufficiently large.

Chuang and Sirbu (1998) introduce two-dimensional heterogeneity of tastes and allow for the possibility of mixed bundling. I adopt these features of their model to study bundling under competition. As Chuang and Sirbu point out, by employing a single variable to model consumer heterogeneity, one can only capture consumers' *aggregate* valuations for a bundle. This is adequate in the pure-bundling context. In the mixed-bundling context, however, it is important to account for the correlation of values across different items as well. For example, mixed bundling may be more profitable relative to pure bundling when one consumer values positively only two of a hundred items in a collection (low correlation) and the other has the same total collection value more or less evenly distributed across fifty of the items (high correlation).

There are two important differences between the assumptions in my work and those in Chuang and Sirbu's study: Chuang and Sirbu assume a positive marginal cost and a single producer. At the center of their study is the question of how the economies of scale and marginal cost affect the relative profitability of pure unbundling, pure bundling, and mixed bundling, with the goal to demonstrate that scholarly journal publishers have incentives to unbundle their journals – i.e., to provide individual articles as well as journals – given how technology has been changing the costs. With a positive marginal cost, it is not surprising that they find that for a monopolist, mixed bundling is more profitable than either of the pure strategies. They also allow readers to value some articles at less than marginal production cost. Thus it is possible for component selling to strictly dominate bundling, since bundling results in the costly distribution of products that have below-cost value to some consumers. In this chapter, I set the marginal cost to zero to study the effects of price discrimination and buyer-diversity reduction of different bundling strategies under competition.

A number of authors have studied bundling in competitive settings. Fishburn, Odlyzko, and Siders (2000) consider a duopoly in which each firm produces an identical set of N information goods (perfect substitutes). By assumption, one firm bundles while the other offers component pricing. In nearly all of their numerical simulations, a price war ensues with both firms' prices falling towards zero (marginal cost).

Matutes and Regibeau (1992) and Farrell, Monroe, and Saloner (1998) consider duopolists that produce complements rather than substitutes. These authors study bundling of two complementary goods that may be purchased from separate firms. Nalebuff (2000) finds that a firm that sells a bundle of complementary products will have a substantial competitive advantage over rivals who sell the component products individually. Some authors examine bundling as a tying strategy when one firm is a monopolist over one product but faces potential competition for a second product (see, e.g., Carbajo et al., 1990; Whinston, 1990; Aron & Wildman, 1999). Nalebuff (1999) explores how bundling can be used as an entry deterrent.

Bakos and Brynjolfsson (1999) consider a duopoly in an N-good market and (stochastically) identical consumers. Goods are pair-wise substitutes: the demand for one good is independent of the demand for all but one of the remaining goods. This drastically restricts the possible strategic interactions. In this chapter, all item valuations are (weakly) subadditive, and thus all items depress one another's value.

There is an extensive body of literature on product differentiation under linear pricing. Goods are assumed to have a fixed and a positive marginal cost of production, and the firms choose their market niche as well as the price of their differentiated good. The linear-city model due to Hotelling (1929), the circular-city model due to Salop (1979), and a symmetric model of monopolistic competition due to Dixit and Stiglitz (1977) and Spence (1976) are examples of classic models of horizontal differentiation. In none of these studies, however, is the model of consumer tastes rich enough to study the questions I pose in this chapter. Hotelling (1929) and Salop (1979) characterize consumers by a single type variable, and consumer valuations of goods are assumed to be independently and identically distributed. As discussed above, one-dimensional heterogeneity of tastes is not adequate in the context of mixed bundling. In the Dixit-Stiglitz-Spence model, there exists a single, representative consumer with a constant-elasticity utility function that depends on the consumption levels of items produced by different sectors (firms). That is, there is no heterogeneity in tastes whatsoever. The rationale for product differentiation stems from the consumer's broad spectrum of interest rather than heterogeneity in the consumer population.

For this chapter, I build directly on the work of Fay and MacKie-Mason (1999) and Fay (2001), Chapter 2. Fay and MacKie-Mason show that under homogeneous consumer preferences, bundling achieves the first-best solution to a firm's profit-maximization problem and unbundling yields less profit than the first-best solution. Under heterogeneous preferences, they find that introducing competition from a second firm results in much lower prices than under monopoly, yet only a moderate profit reduction. However, the latter result may be attributed to the specific properties of the value function used in the analysis of environments with heterogeneous consumers. The value function used to model heterogeneity of consumer tastes had an undesirable property: the value of consuming only firm 2's bundle is positively correlated with the number of items in firm 1's bundle, even if the consumer does not have access to that bundle. The valuation function I use in this study does not have this property.[5] In addition, Fay and MacKie-Mason did not consider mixed bundling and were unable to find any mixed-strategy Bertrand equilibria of the pricing game in which both firms use pure bundling. I overcome these difficulties by using more sophisticated game-solving techniques (see Section 4).

3 Bundle-Pricing Game

3.1 Firms

Two firms each control a collection of items, sized $N_1 > 0$ and $N_2 > 0$, respectively (N_1 and N_2 are fixed and exogenous). An example of such a collection would be a set of different book titles, a collection of different news stories or scholarly articles, a collection of music recordings or movies or software programs. A book title, a news story, or a song would be an example of a single item. There may be substantial sunk costs to create the goods,[6] but the marginal cost of reproducing and distributing is zero. Although in practice marginal cost is rarely strictly zero, the simplifying zero-marginal-cost assumption allows one to explore what happens as marginal costs become vanishingly small, as is typically the case for digital information goods.

I restrict each firms' bundling choices to three *bundling strategies*: selling items individually at the same per-item price, to which I refer as *pure unbundling* (U), *pure bundling* (B) of the whole collection for a single bundle price, and *mixed bundling* (M) that combines the two options.[7] I exclude sub-bundling strategies for reasons of computational tractability. Even for this simplified problem, solving for a mixed-strategy equilibrium sometimes took days, depending on the consumer preference distribution. I analyze this market as a duopoly game in which the two firms simultaneously choose their bundling

[5]The observation about the undesirable property of the valuation function studied by Fay and MacKie-Mason (1999) is due to Scott A. Fay. Fay also proposed the valuation function presented in this chapter.

[6]We can think of first-copy costs as sunk, which allows to treat such costs as exogenous variables in the model. In other words, my model does not encompass the decision to produce the goods.

[7]It is important to distinguish between a mixed-bundling strategy and a *mixed strategy*, where the latter is a term commonly used in game-theoretic literature. Here the mixed-bundling strategy (M) is defined as a pair of prices: a bundle price and a per-item price. This strategy can be *pure* in the sense that a firm can "play" this pair with probability one. For example, a firm can offer a choice between buying the whole collection at \$10 or any subset of items at \$1 per item. A mixed strategy is a probability distribution over a subset of pure strategies. For example, in half of its stores, the firm can offer the whole collection for \$7 and each individual item for \$2 instead of the previous combination. Then the firm's strategy can be viewed as a mixed strategy of two mixed-bundling strategies, (10,1) and (7,2), each chosen with probability $\frac{1}{2}$ at each store.

and pricing strategies. Thus, the pricing strategy of firm i, $i \in \{1, 2\}$, is a per-item price $p_i \geq 0$ under pure unbundling, a bundle price $P_i \geq 0$ under pure bundling, and a pair of prices (p_i, P_i) under mixed bundling.[8]

3.2 Consumers

In this section, I introduce a model of consumer preferences over information goods and define the consumer's choice problem. For the purpose of this study, different book titles are instances of the same information good (books), different software programs are instances of the information good "software", etc. Typically, consumers buy only one copy of a book, software, song, or a news story. I therefore make the following assumption.

Assumption 1.1. *Consumers have demand for at most one unit of a particular item.*

Consumers vary in the quantity of the information good in which they are interested. For example, some consumers may spend all day viewing videos on `youtube.com`, and some view only a couple. On the book market, consumers vary by the number of books they read per year. Consumers also differ in the intensity of their preferences. The preference intensity is high if the consumer has particularly high values over some items available on the market. For example, fans of the interactive web-based video series *lonelygirl15*[9] are an example of consumers with high-intensity preferences over videos. Academic achievement of biology professors depends on their knowledge of the latest biology research, and therefore they have high-intensity preferences over scholarly articles.

Following Chuang and Sirbu (1998), let us assume that the consumers' valuations of individual items in a collection are correlated. The correlation in a collection is defined by two parameters: intensity and breadth. The difference from the Chuang-Sirbu model here is that the consumer can choose from two collections rather than a single one. Let $w_i > 0$, $i \in \{1, 2\}$, be the value of the consumer's most favored item in collection i. This variable describes the intensity, or depth, of the consumer's tastes for collection i. Let $k_i > \frac{1}{N_i}$ represent the preference breadth of a given consumer for collection i. Roughly, k_i can be thought of as the fraction of the items that the consumer is willing to consume from collection i. In fact, this is an accurate interpretation of the parameter if $k_i \leq 1$. If k_i is greater than one, the concept of such a fraction is not well defined. The meaning of k in that case will become clear later. I impose another simplifying assumption on w and k.

Assumption 1.2. *For each consumer, her preference breadth and intensity are the same across different collections when collections are considered independently, i.e.,* $w_1 = w_2$ *and* $k_1 = k_2$.

[8]Note that mixed bundling subsumes both pure schemes: setting p_i to infinity is equivalent to pure bundling, and setting P_i to infinity is equivalent to pure unbundling.

[9]`http://www.youtube.com/user/lonelygirl15`

I will therefore suppress the collection subscript and denote breadth simply by k and intensity by w. Equal breadth implies, for example, that a rock fan likes the same number of songs in two different music collections of the same size. In a collection of twice that size, she will like twice as many items. Equal intensity implies that the consumer is equally happy with her top choices from either collection. It is straightforward to extend the analysis to cases in which $w_1 \neq w_2$ and $k_1 \neq k_2$, but this is outside the scope of this work.

Consumers may place zero value on any number of items. Suppose each consumer has ranked all items by their individual valuations. I label the individual valuation of an item x by $v(x)$. Consider $n_1 < N_1$ highest-ranking items from collection 1 and $n_2 < N_2$ highest-ranking items from collection 2.

Assumption 1.3. *A consumer's preferences are represented by a quasilinear utility function $U(n_0, n_1, n_2) = u(n_0) + V(n_1, n_2)$, where n_0 is the numéraire and $V(n_1, n_2)$ is a* subutility function *that depends on n_1 and n_2, where n_1 and n_2 are as defined above.*

I approximate quantities n_1 and n_2 with continuous variables. Consider first the following subutility function V':

$$\begin{aligned} V'(n_1, n_2) = \max_{m_1, m_2} w\Big(m_1(1 - \frac{m_1}{2kN_1}) + m_2(1 - \frac{m_2}{2kN_2})\Big), \\ \text{s.t.} \quad m_1 \geq 0, \quad m_2 \geq 0, \\ m_1 \leq n_1, \quad m_2 \leq n_2. \end{aligned} \tag{1}$$

I will refer to the function that is maximized as f':

$$f'(x_1, x_2) = w\Big(x_1(1 - \frac{x_1}{2kN_1}) + x_2(1 - \frac{x_2}{2kN_2})\Big). \tag{2}$$

The maximization part in Equation (1) is to ensure that the property of free disposal is satisfied. If the consumption pair (n_1,n_2) is such that the constraints are binding – which is true for (n_1,n_2) where f' is increasing – function V' is simply f'. Otherwise, V' is set to the value achieved at the satiation point, i.e., at the point where f' is maximized.

I now show that V' is a straightforward generalization of the Chuang-Sirbu model to two collections. To obtain the item valuation function, we take the derivative of f' with respect to x_i, $i \in \{1, 2\}$:

$$\frac{\partial f'(x_i, x_j)}{\partial x_i} = w(1 - \frac{x_i}{kN_i}). \tag{3}$$

This is a downward-sloping straight line. Remember that the free-disposal property implies that item valuations cannot be negative. Thus, the valuation $W(n_i)$ of the consumer's n_ith highest-ranking item from collection i becomes

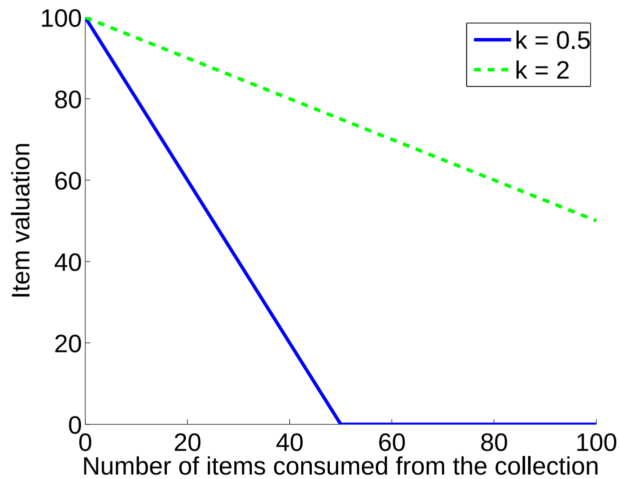

Figure 1: Valuation function $W(n_i)$ for $N_i = 100$, $w = 100$, and two different values of k: $k = 0.5$ and $k = 2$. On the horizontal axis, the items are in the decreasing order of preference. The slopes of the lines are $\frac{-w}{kN_i} = -2$ and -0.5, respectively. The 0.5-k consumer would consume $kN_i = 50$ out of 100 items. The cutoff number 50 is the intersection of the line with the horizontal axis. The rest of the items have zero value. The 2-k line intersects the horizontal axis at $n_i = 200 > N_i$. This consumer consumes all 100 items and has a strictly positive value of the least preferred item.

$$W(n_i) = \max\{0, w(1 - \frac{n_i}{kN_i})\}, \tag{4}$$

where $0 \leq n_i < N_i$.

This is exactly the Chuang-Sirbu valuation function for a single collection. Figure 1 displays $W(n_i)$ for $N_i = 100$, $w = 100$, and two different values of k: $k = 0.5$ and $k = 2$. It also clarifies the interpretation of k when it is greater than one: k defines the slope of the valuation function ($\frac{-w}{kN_i}$).

Another assumption concerns the consumer's behavior when she is indifferent between consuming or not consuming an item. Define marginal surplus of an item as marginal value minus marginal cost of obtaining the item.

Assumption 1.4. *Consumers do not consume items with zero marginal surplus.*

We can see from Equation (4) that the consumer's valuations of items from one collection are not related in any way to her valuations from the other collection. Therefore, with such preferences, we are still looking at the Chuang-Sirbu world, in which the interaction between each firm and consumers can be considered separately: they do not have to compete for consumers. In order to make the market more competitive, I introduce a parameter $\gamma > 0$ and modify the subutility function as follows:

$$V(n_1, n_2) = \max_{m_1, m_2} w\Big(m_1(1 - \frac{m_1}{2kN_1}) + m_2(1 - \frac{m_2}{2kN_2})\Big) - \gamma(m_1 + m_2)^2,$$
$$\text{s.t.} \quad m_1 \geq 0, \quad m_2 \geq 0, \qquad (5)$$
$$m_1 \leq n_1, \quad m_2 \leq n_2.$$

I use V as the consumer's subutility function to study competition. I will refer to the function being maximized as f:

$$f(x_1, x_2) = w\Big(x_1(1 - \frac{x_1}{2kN_1}) + x_2(1 - \frac{x_2}{2kN_2})\Big) - \gamma(x_1 + x_2)^2. \qquad (6)$$

Note that the consumer value of some N items does not depend on which firm or how many firms the items belong to. This property is desirable when seeking to compare different market models. Suppose firm 1 owns a fraction α of the N items, and firm 2 owns the rest of them. Consider a consumer's n highest-ranking items. To simplify exposition, suppose that she values them all positively. Given Assumption 1.2, these n items will correspond to αn and $(1 - \alpha)n$ highest-ranking items in the two collections, respectively. The consumers' subutility then equals

$$\begin{aligned} &f(\alpha n, (1 - \alpha)n) = \\ &w\Big(\alpha n(1 - \frac{\alpha n}{2k\alpha N}) + (1 - \alpha)n(1 - \frac{(1 - \alpha)n}{2k(1 - \alpha)N})\Big) - \gamma(\alpha n + (1 - \alpha)n)^2 = \\ &w\Big(n(1 - \frac{n}{2kN})\Big) - \gamma n^2, \end{aligned} \qquad (7)$$

which is the consumer's subutility of n highest-ranking items from a single collection of size N. The more general case when some of the n items have zero value to the consumer can be proved similarly: those items have no effect on the utility.

With $\gamma > 0$, the consumer's valuations of positively valued items from different collections become *strongly subadditive*, as opposed to additive. Goods are subadditive if the combined set is worth less than the sum of its parts. This links the firms' markets in the following way: the price a consumer is willing to pay for an item depends on the number of items consumed from the other collection, which in turn depends on the other firms prices. In other words, firm i's prices affect the demand for firm j's goods.

To see that the parameter γ introduces subadditivity, it is easier to work with f. Note that the cross-derivative of f is negative for positive γ:

$$\frac{\partial^2 f(x_1, x_2)}{\partial x_1 \partial x_2} = -2\gamma. \qquad (8)$$

This implies that for positively valued items, the marginal valuation of the n_ith preferred item from collection i decreases as the number of items n_j consumed from collection j increases, $i, j \in \{1, 2\}$. Decreasing marginal valuation is equivalent to subadditivity (?).

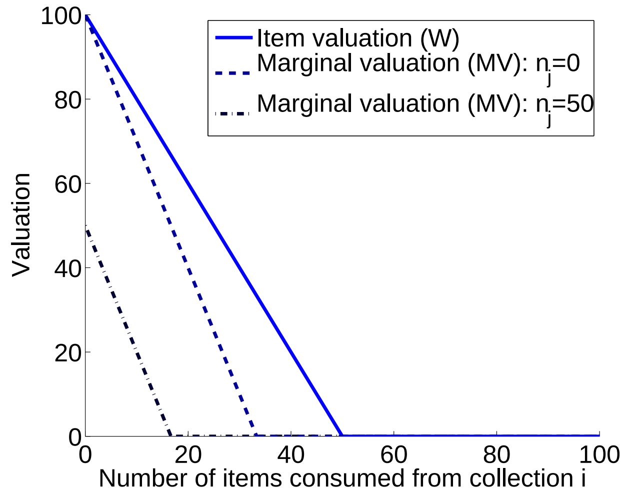

Figure 2: Valuation function $W(n_i)$ and marginal-valuation function $MV(n_i)$ for $N_i = 100$, $w = 100$, $\gamma = 0.5$, and two different values of n_j: $n_j = 0$ and $n_j = 50$. On the horizontal axis, the items are in the decreasing order of preference. The slope of the W line is $\frac{-w}{kN_i} = -2$. The slopes of the MV lines are $\frac{-w}{kN_i} - 2\gamma = -3$. MV for $n_j = 50$ is shifted down by $2\gamma n_j = 50$ units.

It is important to mention that γ also introduces subadditivity between items from the *same* collection. To see that, let us calculate the marginal valuation function of the consumer's n_ith highest-ranking item from collection i for a given number of items n_j from collection j $(i, j \in \{1, 2\})$:

$$MV(n_i) = \max\{0, w(1 - \frac{n_i}{kN_i}) - 2\gamma(n_i + n_j)\}, \tag{9}$$

where $0 \leq n_i < N_i$, and n_j is fixed at some number between 0 and N_j.

Figure 2 displays $W(n_i)$ and $MV(n_i)$ for $N_i = 100$, $w = 100$, $k = 0.5$, and $\gamma = 0.5$ at two different values of n_j: $n_j = 0$ and $n_j = 50$. Note how subadditivity between items from the same collection reduces the slope of the marginal-valuation line (by 2γ) and how subadditivity between items from different collections shifts the marginal-valuation line for collection i down by $2\gamma n_j$.

By assumption, the term $\frac{-w}{kN_i}$ is the slope of the item valuation function W, which describes the valuation of each item given no other items have been consumed. If $\gamma = 0$, the marginal-valuation function equals W. If $\gamma > 0$, the slope becomes steeper. That is, the marginal value of consuming an item is now below its valuation, and it is decreasing.[10] Therefore, the valuations of items from the same collections are subadditive if $\gamma > 0$. One

[10]Implicitly in this argument, I rely on Assumption 1.5, which I introduce at the end of this section. Given that the valuation function V and MV are defined for preference-ordered items, the conclusion does not directly follow if the items in the domains of MV and W have different orders.

consequence of this γ-effect is that the firms face a less elastic demand relative to the Chuang-Sirbu model, given the same k and w.

Let us now consider the consumer's choice problem when she faces two competing firms. Given the firms' pricing schemes and particular prices, consumers choose n_1 and n_2 to maximize their surplus (i.e., value minus the cost of acquiring the items). Remember that the pricing strategy of firm $i \in \{1, 2\}$, is a per-item price $p_i \geq 0$ under pure unbundling, a bundle price $P_i \geq 0$ under pure bundling, and a pair of prices (p_i, P_i) under mixed bundling. In the most general case when the firms each offer a mixed bundle, each consumer has the following options: she can buy two bundles; buy only one bundle from one firm and zero or more individual items from the other; buy no bundles at all and zero or more individual articles from both firms. Let d_i, $i \in \{1, 2\}$, equal 1 if the consumer buys firm i's bundle, and equal 0 otherwise. Then the consumer's surplus (CS) of consuming n_1 and n_2 highest-ranking items from collections 1 and 2, respectively, is given by

$$CS(n_1, n_2) = V(n_1, n_2) - (1 - d_1)p_1n_1 - d_1P_1 - (1 - d_2)p_2n_2 - d_2P_2. \tag{10}$$

To find the consumer's optimal choice, we need to maximize CS subject to a number of constraints. Before stating the maximization problem, let me note that V is maximized whenever $f(n_1, n_2)$ is maximized, by definition of V (see Equations (5)–(6)). Other V-maximizing choices contain items with zero marginal values in addition to the f-maximizing set. According to Assumption 1.4, consumers do not consume items with zero marginal surplus. Therefore, they will never consume items with zero marginal value. Then the number of consumed items n_1 and n_2 will always be below or at the satiation point, and the consumer optimization problem can be written as follows:

$$\begin{aligned} \max_{n_1,n_2} \Big(f(n_1, n_2) - (1 - d_1)p_1n_1 - d_1P_1 - (1 - d_2)p_2n_2 - d_2P_2 \Big), \\ \text{s.t.} \quad n_1 \geq 0, \quad n_2 \geq 0, \\ n_1 < N_1, \quad n_2 < N_2. \end{aligned} \tag{11}$$

We need an additional assumption to make the consumer's choice well defined. Recall that the domain of the subutility function V is a rank-ordered set of items, with the items ranked according to their individual valuations $v(x)$. Suppose for a moment that we are back to one collection on the market. Given that the per-item price is the same for all items, consumers never have to consider item subsets other than those defined by the number of top-ranking items, when buying items individually. For example, if $v(x) > v(y) > v(z)$ for a particular consumer, then this consumer never has to consider the set $\{x, z\}$ to make the optimal choice, because it would never be optimal to exclude item y when choosing to consume z. Similarly, if a consumer buys a full collection as a bundle, the marginal cost of consuming each item is zero, and therefore the above argument applies. When there are two collections on the market, an additional assumption is required to ensure that a consumer will never want to consume a set of n items from collection i that is not the first n top-ranked items from i.

To see that, consider the following example. Suppose there are two collections (collection 1 and 2), each consisting of one economics article (E_i), one computer science article (CS_i), and one political science article (PS_i). Suppose also that the consumer is generally more interested in economics than in computer science and more in computer science than in political science. Let us extend the definition of $v(X)$ to be the consumer's valuation of *any* article subset X. For concreteness, suppose $v(E_1) = 30$, $v(E_2) = 25$, $v(CS_1) = 20$, $v(CS_2) = 15$, $v(PS_1) = 10$, $v(PS_2) = 5$. Moreover, the consumer prefers to read broadly, but does not have time to read more than one article in each category. More specifically, articles from the same fields have zero marginal values (subadditive preferences), while articles from different fields are additive: $v(\{E_1, E_2\}) = 30$, $v(\{CS_1, CS_2\}) = 20$, $v(\{PS_1, PS_2\}) = 10$; let i, j, h can be 1 or 2, then $v(\{E_i, CS_j\}) = v(E_i) + v(CS_j)$, and similarly for all other pairs of articles from different fields; and finally, $v(\{E_i, CS_j, PS_h\}) = v(E_i) + v(CS_j) + v(PS_h)$. Suppose now that the consumer is considering buying CS_1. Then the marginal value of CS_2 is zero, and the consumer may want to consider the subset $\{E_2, PS_2\}$ from collection 2. However, the subutility function V is not defined for this subset. It is only defined for the following subsets from collection 2: $\{E_2\}$, $\{E_2, CS_2\}$, and $\{E_2, CS_2, PS_2\}$. I therefore impose the following assumption on the marginal valuations.

Let $v(A)$ be the consumer's value of *any* set of items A. Let $mv(x|A)$ be the consumer's marginal value of item x in the set $\{x \cup A\}$.

Assumption 1.5. *Suppose $v(x) > v(y)$, where x and y are two single items. Then for any set A, $mv(x|A) > mv(y|A)$.*

This assumption is sufficient to ensure that no matter what items a consumer chooses to consume from one collection, the order of items from the other collection ranked by marginal valuations would be the same as the order by v. This in turn ensures that the consumer's choice problem is always well defined, even though V is undefined for some subsets of items.

4 Empirical Game Analysis

Recall that the purpose of this work is to analyze competition of bundling firms when consumers have heterogeneous tastes. Heterogeneity of the consumer population is defined by the distribution of two parameters, w and k introduced in Section 3.2. I have not been able to solve analytically for Nash equilibrium strategies in this game for the case of general distributions of w and k. As an alternative approach, I employ the *empirical game-theoretic* methodology discussed in Chapter **??**. In particular, for each environment described in the following section, I use computer simulation to estimate the demand for a variety of the firms' strategy profiles. I explain the simulation procedure in Section 4.1. Given the demand, I calculate the firms' expected profits for each strategy profile. Then I solve for (mixed) Nash equilibria of the *restricted* competition game.[11]

To study relative performance in equilibrium of mixed bundling, pure bundling and pure unbundling, in addition to the unconstrained mixed-bundling game, I analyze all possible subgames in which at least one firm is restricted to either B or U. I label them as X_1X_2, where $X_1, X_2 \in \{\text{B, U, M}\}$ denote the bundling-strategy space to which firm 1 and firm 2, respectively, are restricted. There are eight such subgames: BB, BU, UB, BM, MU, UU, UM, MU; and the ninth possible game is the unrestricted MM.

4.1 Environments and Strategy Space

Particular environments are defined by specifying the market model (number of firms and collection sizes), and a preference model comprising the value of the substitution parameter γ and the probability distributions over parameters w and k in the population of consumers. I have analyzed a total of nine environments: for each of the three preference models described in Table 1, I analyzed three market models (Table 2).

The full strategy profile space is infinite ($\mathbb{R}^4_+$). To make the problem manageable, I limit sampling range to the following price intervals: $P_i \in (0, 1920]$ and $p_i \in (0, 120]$. The upper bounds on the price intervals were chosen based on a few rounds of preliminary simulations with $P_i \leq 2400$ and $p_i \leq 120$, for which all pricing strategies that survived iterated elimination of strongly dominated strategies (IESDS) were such that $P_i \leq 1800$ and $p_i \leq 115$. Similarly, zero price did not survive IESDS in any of the preliminary simulations, and therefore was not included in the intervals. For each of the nine environments, I evaluated the nine subgames (BB, BU, UB, BM, MU, UU, UM, MU, and MM) sampling prices within the same range and at the same constant intervals:

$$\begin{aligned} p_i &\in \{4, 8, 12, \ldots, 120\}, \\ P_i &\in \{40, 80, 120, \ldots, 1920\}. \end{aligned} \tag{12}$$

Thus, each firm's strategy space consists of 48 strategies in the BB game, 30 strategies in the UU game, and 1440 (48×30) strategies in the MM game. The interval sizes (40 for P_i and 4 for p_i) were primarily dictated by computational considerations. For example, it

[11] To find Nash equilibria I used Gambit, a library of game theory software and tools for the construction and analysis of finite extensive and normal form games (?). In particular, I used the `gambit-lcp` algorithm for solving two-player nonzero-sum games (documentation available at `http://gambit.sourceforge.net/doc/gambit-0.2007.01.30/gambit/`). This algorithm does not necessarily find all equilibria.

took about 2 days to do payoff estimation for each of the MM subgames for the preference model P3 (see Table 1 below), and about 5 more days for Gambit to find 189 equilibria for one of them. I have also analyzed the following larger "refined" strategy spaces:

$$\begin{aligned} p_i &\in \{4, 8, 12, \ldots, 120\}, \\ P_i &\in \{20, 40, 60, \ldots, 1920\}; \end{aligned} \tag{13}$$

$$\begin{aligned} p_i &\in \{2, 4, 6, \ldots, 120\}, \\ P_i &\in \{40, 80, 120, \ldots, 1920\}; \end{aligned} \tag{14}$$

$$\begin{aligned} p_i &\in \{2, 4, 6, \ldots, 120\}, \\ P_i &\in \{20, 40, 60, \ldots, 1920\}; \end{aligned} \tag{15}$$

$$\begin{aligned} p_i &\in \{3, 6, 9, \ldots, 120\}, \\ P_i &\in \{30, 60, 90, \ldots, 1920\}. \end{aligned} \tag{16}$$

For these strategy spaces, I was not able to solve all of the nine subgames. In particular, the MM game in the P3-SD environment (see Tables 1–2 below) was too large for Gambit to solve in less than a week. Also, it took prohibitively long to solve the MM subgame (for which all equilibria I could find are strictly mixed-strategy equilibria with relatively large supports) in any of the environments for (15). Since the MM subgame is central to the analysis of mixed bundling, I report results only for the strategy space (12), for which I was able to solve all the subgames of interest. These results are not significantly different from those obtained for the finer strategy sets: in all the equilibria I could find for (13)–(16), the equilibrium price ranges significantly overlap[12] and the equilibrium payoff of any player lies within –4% to +1% of the corresponding equilibrium payoff for the strategy space (12).

PID	w	k
P1	100	Exp(6.568)
P2	U[0,200]	0.152
P3	U[0,200]	Exp(6.568)

Table 1: List of preference models. In all models $\gamma = 0.5$. The choice of preference distributions and the specific distribution parameters is motivated by empirical studies by King and Griffiths (1995) (see Appendix A).

Table 1 summarizes the preference models I have analyzed. Model P3, in which both w and k have non-degenerate independent distributions, is the closest to the Chuang-Sirbu model.[13] In models P1 and P2, one of the parameters is constant. These models allow

[12] In all cases of multiple equilibria, the equilibrium price ranges always significantly overlap within the multiple-equilibrium set.

[13] They assume $\gamma = 0$, w is uniformly distributed between 0 and 1, and k is distributed exponentially with $\lambda = 13.8758$.

MID	N_1	N_2	Number of firms	Description
SD	100	100	2	Symmetric duopoly
ND	50	150	2	Non-symmetric duopoly
M	200	n/a	1	Monopoly

Table 2: List of market models.

me to analyze the effect of competition for each type of the corresponding one-dimensional heterogeneity type.

For each preference model, I analyze three market structures given in Table 2. I label preference models and market models by their ID (PID and MID, respectively) and each environment as a pair PID-MID. Note that the total number of items on the market is 200 for all market models. To study the effect of competition, I compare the properties of the equilibrium outcomes of two duopoly models (one symmetric and one non-symmetric) to those of the optimal monopoly strategy.

The choice of preference distributions and the specific distribution parameters is motivated by empirical studies of distribution of scholarly articles performed by King and Griffiths (1995). Their results suggest that the parameter k follows an exponential distribution in the population of readers that the authors sampled (see Appendix A). Using the empirical data, I fit k to an exponential distribution with the parameters specified in preference model P1. See details in Appendix B. For the alternative heterogeneity model P2, I fix the breadth at the mean of the exponential distributions of model P1 ($k = 0.152$). I chose the range of w to be such that the mean intensity is equal to the fixed intensity value in model P1 ($\mathrm{E}(w) = 100$).

To estimate the firms' expected profits, I used the following procedure. For the preference model P2, in which $w \sim \mathrm{U}[0,200]$, I computed the demand of all consumers with integer $w \in \{1, \ldots, 200\}$. Below I refer to these types as w_g, where $g \in \{1, \ldots, 200\}$. For the model P1, in which $k \sim \mathrm{Exp}(6.568)$, I computed the demand for 284 consumers with k-types given by $k_{q+1} = k_q + 0.001\delta^{q-1}$ for $q = \{1, \ldots, 284\}$, where $k_0 = 0$ and $\delta = 1.03$. The 284th type is $k_{284} = 147.4186$. For the model P3, I computed the demand of 56800 (200×284) consumers with all combinations of w_g and k_q used in models P1 and P2. Given the demand, I computed the profit, $\pi_i(w_g, k_q)$, that each firm i would earn from each estimated consumer type (w_g, k_q). The estimated expected profit of firm i, $i \in \{1, 2\}$ is given by

$$\mathrm{E}\pi_i = \sum_{\substack{g=\{1,\ldots,200\} \\ q=\{1,\ldots,284\}}} \mathrm{P}(w_{g-1} < W < w_g)\mathrm{P}(k_{q-1} < K < k_q)\pi_i(w_g, k_q), \tag{17}$$

where $w_0 = 0$, $k_0 = 0$, and W and K are random variables following the distributions from

Table 1.

Such procedure appears to produce rather accurate profit estimates. For the UU subgame under P3-preferences and three market models – SD, ND, and M – I estimated the equilibrium (optimal) prices and profits analytically.[14] For the M-model, I also found the optimal price and payoff for the case when the monopoly is restricted to pure bundling. For the five equilibrium (optimal) payoff estimates obtained in these four games,[15] the numerically estimated profits are all systematically lower, by at most 1.5%, than the theoretical estimates. The equilibrium (optimal) prices are within 6% of the theoretical estimates (lower by up to 4% and higher by up to 6%).

4.2 Empirical Results

In Table 3, I report for the duopoly market models the number of strategies that survived the iterated elimination of strongly dominated strategies (IESDS) in each of the nine subgames. Games with many survivors tend to have multiple strictly mixed-strategy equilibria. The largest number of multiple equilibria in a game is 189 and the second largest is 3. All multiple equilibria have very similar payoffs (see Table 4). Therefore, in the case of multiple equilibria, I report only one of them, giving priority to symmetric Pareto superior equilibria.

The largest support of a single mixed-strategy equilibrium includes 26 strategies for each firm, but most mixed-strategy equilibria do not have more than 5–10 strategies in the support. See Tables 5–7 for the supports of representative equilibria in each subgame. In the tables, a row and a column define a subgame. In each cell, the top number(s) is the equilibrium price (price pair) of the row firm, and the bottom number(s) is the equilibrium price (price pair) of the column firm. I denote a mixed-bundling price pair as p-P. For example, in the UM subgame (row U and column M) for the symmetric duopoly under the P1-preference distribution (Table 5, P1-SD), firm 1's equilibrium (per-item) price is 40, and firm 2's equilibrium (mixed-bundling) price pair is 60-720, which means that it offers a choice between buying individual items for 60 each and buying the whole collection for 720. For large supports, I report ranges of prices. For example, "10 in [60,80]×[440,480]" means that there are 10 price pairs in the support of this equilibrium mixed-bundling strategy, and the first component – the per-item price – varies from 60 to 80, and the second component – the bundle price – varies from 440 to 480.

The fact that many equilibria involving bundling are strictly mixed raises the question of their practical implementation and interpretation. In the context of information-good production, the support of mixed-strategy equilibria can be interpreted as a near-

[14] I did not impose the constraints that the quantity consumed by each consumer type from a collection is less than the collection size. Therefore, the analytically computed profits may be overestimated.

[15] In the non-symmetric duopoly, the two firms have different equilibrium (optimal) payoffs and prices.

Symmetric Duopoly				Non-Symmetric Duopoly			
P1-SD				P1-ND			
	Firm 2(1)				Firm 2 (the bigger)		
Firm 1(2)	U	B	M	Firm 1	U	B	M
U	1 1			U	1 1	1 1	2 2
B	1 1	1 1		B	1 1	1 1	2 4
M	1 1	5 4	26 26	M	1 1	6 5	8 11
P2-SD				P2-ND			
	Firm 2(1)				Firm 2 (the bigger)		
Firm 1(2)	U	B	M	Firm 1	U	B	M
U	1 1			U	1 1	1 1	2 2
B	1 1	6 6		B	1 1	2 6	2 9
M	1 1	35 6	47 47	M	1 1	13 6	14 21
P3-SD				P3-ND			
	Firm 2(1)				Firm 2 (the bigger)		
Firm 1(2)	U	B	M	Firm 1	U	B	M
U	1 1			U	1 1	1 1	1 1
B	1 1	7 7		B	1 1	2 6	1 1
M	1 1	38 7	116 116	M	1 1	7 5	26 37

Table 3: Number of survivors of iterated elimination of strongly dominated strategies (IESDS). A row and a column define a subgame. In each cell, the first number is the number of survivors for the row firm, and the second number is for the column firm. For example, in the MM subgame (row M and column M) in environment P1-ND, 8 strategies remained in the smaller (row) firm's strategy set, and 11 remained in the bigger (column) firm's strategy set. Each firm's original strategy space consists of 48 strategies in the BB game, 30 strategies in the UU game, and 1440 (48×30) strategies in the MM game.

Environment	Subgame	# of Multiple Eq.	Profit Diff. (%)
P1-SD	MM	3	0.02
P2-SD	BB	3	1.68
P3-SD	BB	3	0.57
P3-SD	MM	189	0.32

Table 4: Subgames in which Gambit found multiple equilibria. In the last column I report the (absolute) difference in the payoff of the unreported equilibria as a percentage of the reported equilibrium payoff.

equilibrium price region. Note that in Table 6, which describes the non-symmetric duopoly, the sizes of the supports are all less than three, and the prices are rather close. Since firms are never perfectly symmetric, this suggests that in practice such price regions may be small. One example when a randomized choice from these regions may arise naturally is during a readjustment to some external shock, when both firms need to react immediately and simultaneously.

I first analyze the relative performance of different duopoly equilibrium pricing schemes, or the optimal prices in case of a monopoly. Table 8 presents equilibrium profits for the duopoly subgames in each of the environments. Table 9 presents total industry profits for duopolies as well as the monopoly. We can see that the relative payoffs of mixed bundling compared to pure unbundling and pure bundling depend on the type of consumer heterogeneity. The differences in profits are more clearly seen from Table 10. For the monopoly, we have the following result (see the last column in Table 10).

(i) Mixed bundling yields higher profits relative to pure bundling when breadth varies exponentially across consumers (preference models P1 and P3): pure bundling yields 80–83% of the mixed-bundling profits according to the empirical results.

(ii) Pure bundling is dominated by pure unbundling when breadth varies across consumers (models P1 and P3): the empirical analysis shows that pure bundling yields 80% the mixed-bundling profit vs. 90% produced by pure unbundling in P1; 83% vs. 99.2% in P3.

(iii) Pure bundling is as profitable as mixed bundling when the breadth is the same for all consumers, while the intensity of their preferences vary (model P2).

(iv) Pure unbundling is almost as profitable as mixed bundling when both breadth and intensity vary (model P3): it yields 99.2% of the mixed-bundling profits according to the empirical results.

(v) When one of the preference parameters is constant across consumers (models P1 and P2), mixed bundling yields higher profits relative to pure unbundling: the latter yields 85–90% of the mixed-bundling profits according to the empirical analysis.

We already know from Chuang and Sirbu (1998) that pure unbundling can dominate pure bundling even in the presence of (weak) economies of scale and that mixed bundling strictly dominates the two when the marginal cost is positive. The authors conclude that "the choice of optimal bundling strategy lies in the balance between cost-savings from bundling and loss of surplus due to exclusion violation,"[16] where exclusion violation is

[16] Chuang and Sirbu (1998), p. 13.

Symmetric Duopoly			
P1-SD			
	Firm 2(1)		
Firm 1(2)	U	B	M
U	44 44	40 480	40 60-720
B	480 40	360 360	400 440 56-520 64-400
M	60-720 40	56-520 64-400 400 440	10 in [60,80]×[440,480] 10 in [60,80]×[440,480]
P2-SD			
	Firm 2(1)		
Firm 1(2)	U	B	M
U	52 52	52 640	52 100-640
B	640 52	5 in [400×560] 5 in [400×560]	5 in [400,560] 5 in [72,96]×[400,560]
M	100-640 52	5 in [72,96]×[400,560] 5 in [400,560]	5 in [72,96]×[400,560] 5 in [72,96]×[400,560]
P3-SD			
	Firm 2(1)		
Firm 1(2)	U	B	M
U	52 52	48 640	48 60-960
B	640 48	5 in [320,520] 5 in [320,520]	5 in [440,600] 5 in [56,76]×[440,920]
M	60-960 48	5 in [56,76]×[440,920] 5 in [440,600]	26 in [56,88]×[440,920] 26 in [56,88]×[440,920]

Table 5: Strategies in the support of representative equilibria: Symmetric duopoly. A row and a column define a subgame. In each cell, the top number(s) is the equilibrium price (price pair) of the row firm, and the bottom number(s) is the equilibrium price (price pair) of the column firm. I denote a mixed-bundling price pair as p-P. For example, in the UM subgame (row U and column M) under the P1-preference distribution (P1-SD), firm 1's equilibrium (per-item) price is 40, and firm 2's equilibrium (mixed-bundling) price pair is 60-720, which means that it offers a choice between buying individual items for 60 each and buying the whole collection for 720. For large supports, I report ranges of prices. For example, "10 in [60,80]×[440,480]" means that there are 10 price pairs in the support of this equilibrium mixed-bundling strategy, and the first component – the per-item price – varies from 60 to 80, and the second component – the bundle price – varies from 440 to 480.

Non-Symmetric Duopoly			
P1-ND			
	Firm 2 (the bigger)		
Firm 1	U	B	M
U	44	36	36 40
	48	680	60-920 60-960
B	240	160	240 280
	44	600	60-840 60-880
M	56-360	56-240 64-200 76-200	60-240 64-240 68-280
	44	600 640 720	60-840 60-880 68-760
P2-ND			
	Firm 2 (the bigger)		
Firm 1	U	B	M
U	52	48	48 52
	56	920	84-960 88-920
B	280	240 280	240 280
	56	800 960	92-880 96-960
M	80-280	80-240 88-280	64-240 80-240 88-280
	56	800 960	92-800 92-880 96-960
P3-ND			
	Firm 2 (the bigger)		
Firm 1	U	B	M
U	48	44	44
	56	920	64-1560
B	320	200 240	320
	56	720 840	60-1520
M	60-440	76-240 80-240	64-360 68-320 72-360
	52	720 760	60-1440 68-1040 72-1040

Table 6: Strategies in the support of representative equilibria: Non-symmetric duopoly. A row and a column define a subgame. In each cell, the top number(s) is the equilibrium price (price pair) of the row firm, and the bottom number(s) is the equilibrium price (price pair) of the column firm. I denote a mixed-bundling price pair as p-P. For example, in the UM subgame (row U and column M) under the P1-preference distribution (P1-ND), firm 1's equilibrium strategy is a probability distribution over two (per-item) prices: 36 and 40; firm 2's equilibrium strategy is a probability distribution over two mixed-bundling price pairs: 60-920 and 60-960. The first price pair means that the firm offers a choice between buying individual items for 60 each and buying the whole collection for 920. Similarly for the second price pair.

Monopoly			
	P1-M	P2-M	P3-M
U	48	60	60
B	1040	1280	1320
M	60-1360	108-1280	68-1920

Table 7: Optimal strategies: Monopoly. The rows are the pricing schemes, and the columns are the preference models. For example, in row U, column P1-M, the monopoly is assumed to be restricted to pure unbundling and the consumers' preference distribution is described by the model P1. I denote a mixed-bundling price pair as p-P. For example, 60-1360 in row M, column P1-M, means that the monopoly offers a choice between buying individual items for 60 each and buying the whole collection for 1360.

the inefficiency that arises from consumption at submarginal-cost levels. The result above shows that another important factor is the distribution of consumer types. Moreover, as the transaction and distribution costs of digital information goods become negligibly small, the consumer side of the problem caries increasingly more weight.

The result underscores the intuition Chuang and Sirbu offered to motivate their two-dimensional preference model for studying mixed bundling: If the correlation between items (modeled as breadth here) within a collection is the same across consumers, as in model P2, then consumer reservation prices adequately capture the diversity in consumer tastes, and the monopolist can capture as much surplus through pure bundling as through mixed bundling. This is consistent with earlier studies that show that pure bundling allows a monopolist to extract all consumer surplus by reducing buyer diversity, when consumers have i.i.d. valuations over items (e.g., Bakos & Brynjolfsson, 1998). When correlation varies widely, however, as is often the case for scholarly journals, music CDs, TV channels, pure bundling is strictly dominated not only by mixed bundling, but can also be dominated by pure unbundling (models P1 and P3 here).

Under competition, we are interested in the equilibrium performance of different schedules. In Table 10, I present each firm's expected equilibrium profit as a percentage of their profit in the subgame in which they employ mixed bundling, while the other firm's scheme is fixed. We can see that the pattern is similar to that in the monopoly case. Particular numbers differ, but the relative profit sizes are largely the same. The only exception is the non-symmetric duopoly under P1-preferences: for the smaller firm, the relative profitability of U and B in equilibrium is reversed when the other (bigger) firm uses bundling in the pure or mixed form. This suggests that under competition, each pricing scheme works largely in the same way to extract consumer surplus as under monopoly.

In some environments, however, mixed bundling leads to more aggressive price

Symmetric Duopoly							Non-Symmetric Duopoly						
	P1-SD							P1-ND					
	Firm 2(1)							Big Firm					
F. 1(2)	U		B		M		Small	U		B		M	
U	244	244	204	214	216	269	U	125	367	94	322	99	406
B	214	204	204	204	212	246	B	107	335	103	308	114	388
M	269	216	246	212	247	247	M	137	344	119	316	132	388
	P2-SD							P2-ND					
	Firm 2(1)							Big Firm					
F. 1(2)	U		B		M		Small	U		B		M	
U	250	250	247	296	247	296	U	127	376	122	439	123	441
B	296	247	264	264	263	262	B	150.6	368	154	415	156	415
M	296	247	262	263	262	262	M	151.1	368	154	415	155	415
	P3-SD							P3-ND					
	Firm 2(1)							Big Firm					
F. 1(2)	U		B		M		Small	U		B		M	
U	217	217	194	189	205	222	U	112	324	94	277	104	329
B	189	194	160	160	181	194	B	100	305	93	255	101	311
M	222	205	194	181	196	196	M	115	311	99	261	113	313

Table 8: Duopoly expected equilibrium profits. A row and a column define a subgame. In each cell, the left number is the equilibrium expected profit of the row firm, and the right number is that of the column firm. For example, in the MM subgame (row M and column M) in environment P1-ND, the smaller (row) firm's equilibrium profit is 132, and the bigger (column) firm's equilibrium profit is 388.

Symmetric Duopoly				Non-Sym. Duopoly				Mon.	
P1-SD				P1-ND				P1-M	
	Firm 2(1)				Big Firm				
Firm 1(2)	U	B	M	Small	U	B	M		
U	488	419	485	U	493	416	506	U	495
B	419	408	458	B	443	411	502	B	437
M	485	458	493	M	481	435	520	M	548
P2-SD				P2-ND				P2-M	
	Firm 2(1)				Big Firm				
Firm 1(2)	U	B	M	Small	U	B	M		
U	500	543	543	U	503	561	564	U	505
B	543	528	526	B	519	569	571	B	592
M	543	526	525	M	519	569	570	M	592
P3-SD				P3-ND				P3-M	
	Firm 2(1)				Big Firm				
Firm 1(2)	U	B	M	Small	U	B	M		
U	435	383	427	U	436	370	434	U	440
B	383	320	375	B	405	349	413	B	367
M	427	375	390	M	426	360	426	M	444

Table 9: Total expected (equilibrium/optimal) profit. For the duopoly models, a row and a column define a subgame. For example, the number in a row U and a column M is the total expected equilibrium profit when the row firm is restricted to unbundling its items and the column firm can offer a mixed bundle (subgame UM).

Symmetric Duopoly				Non-Symmetric Duopoly								Mon.	
P1-SD				P1-ND								P1-M	
	Firm 2				Big Firm				Small Firm				
F. 1	U	B	M	Small	U	B	M	Big	U	B	M		
U	91	83.1	87	U	91	79	75	U	90	86	89	U	90
B	80	82.9	86	B	78	87	86	B	79	79	81	B	80
P2-SD				P2-ND								P2-M	
	Firm 2				Big Firm				Small Firm				
F. 1	U	B	M	Small	U	B	M	Big	U	B	M		
U	85	94	94	U	84	79	80	U	85	89	89	U	85
B	100.0	100.6	100.4	B	99.7	100.0	100.4	B	99.7	99.9	99.9	B	100.0
P3-SD				P3-ND								P3-M	
	Firm 2				Big Firm				Small Firm				
F. 1	U	B	M	Small	U	B	M	Big	U	B	M		
U	98.1	99.8	104.9	U	97	95	93	U	98.5	97.9	99.6	U	99.2
B	85	82	93	B	87	94	90	B	84	82	84	B	83

Table 10: Total expected (equilibrium/optimal) profit in percent of the mixed-bundling profit. For the duopoly models, the total expected equilibrium profit in the UX and BX subgames, where X can be U, B, or M, is given as a percentage of the equilibrium profits in the MX game.

competition, which overrides its benefits as a price-discrimination mechanism. In such environments, pure schemes yield higher equilibrium profits than the mixed-bundling scheme, although not by much. For example, in the case of a symmetric duopoly under P2-preferences, the equilibrium in the BB subgame is better for both firms than the equilibrium in the UUand MM games (see Table 8, environment P2-SD). Moreover, once in that equilibrium, neither firm would benefit from unilaterally changing its price schedule: the resulting equilibrium prices would produce lower profits for the deviating firm. In fact, the mixed bundling scheme is weakly "dominated" by pure bundling, in the sense that it always leads to weakly lower profits after prices reach an equilibrium given the schedules, although the potential loss is below 1%. Note that this holds only in the environment in which mixed bundling does not offer an advantage as a price-discriminating tool even under monopoly, i.e., when the preference breadth is constant in the population.

If breadth varies as well as depth, the equilibrium in the UU subgame for the symmetric duopoly (environment P3-SD) is more profitable for both firms than that in MM. In this case, however, the firms would want to unilaterally expand their scheme to mixed bundling. Interestingly, the other firm would not follow the move, because that would decrease both firms' profits by around 5% after the price equilibrium is re-established (see Table 10: in the UM game, firm 1 earns 104.9% of its profit in the MM game). This pattern is observed in the environment in which pure unbundling is almost as profitable as mixed bundling under monopoly, while pure bundling yields only 83% of the mixed-bundling profit. That is, when the population distribution is such that bundling fails to reduce buyer diversity while individual-article sales capture almost all surplus a monopoly could capture. Again, increased price competition from utilizing a mixed-bundling scheme hurts profits in an environment in which mixed bundling extracts little more surplus relative to one of the pure forms. Under P1-preferences, when mixed bundling has clear benefits for a single firm, the price competition that corresponds to $\gamma = 0.5$ is not strong enough to override those benefits.

The assumption of the two-dimensional heterogeneity of consumer tastes is key to these results: if the consumers are *ex ante* homogeneous, pure bundling achieves the first-best solution under monopoly (Bakos & Brynjolfsson, 1998) as well as under duopoly (Fay & MacKie-Mason, 1999, Proposition 1), while pure unbundling yields less profit than the first-best solution (Fay & MacKie-Mason, 1999, Proposition 2).

Comparing the duopoly total expected profits to those of a monopoly (see Table 11), we see that competition reduces the industry-wide profits by up to 21% depending on the preference model and the combination of pricing schemes the firms employ. Table 8 shows that the firms' shares of those profits are roughly proportional to their collection sizes. If we compare by pricing schemes, the effect of competition on profits of bundling firms is far greater than on firms relying on sales of individual items. In the worst case, the latter earn

Symmetric Duopoly				Non-Sym. Duopoly				Largest Diff.	
P1-SD				P1-ND				Diagonal	Overall
	Firm 2(1)				Big Firm				
F. 1(2)	U	B	M	Small	U	B	M		
U	-1	-15	-11	U	0	-16	-8	-1	
B	-15	-7	-16	B	-11	-6	-8	-7	
M	-11	-16	-10	M	-12	-21	-5	-10	-21
P2-SD				P2-ND					
	Firm 2(1)				Big Firm				
F. 1(2)	U	B	M	Small	U	B	M		
U	-1	-8	-8	U	0	-5	-5	-1	
B	-8	-11	-11	B	-12	-4	-4	-11	
M	-8	-11	-11	M	-12	-4	-4	-11	-12
P3-SD				P3-ND					
	Firm 2(1)				Big Firm				
F. 1(2)	U	B	M	Small	U	B	M		
U	-1	-13	-4	U	-1	-16	-2	-1	
B	-13	-13	-15	B	-8	-5	-7	-13	
M	-4	-15	-12	M	-4	-19	-4	-12	-19

Table 11: Difference between expected monopoly profits and total expected equilibrium profits in percent of monopoly profits. A row and a column define a duopoly subgame. For example, the profit of the subgame UB is at the intersection of row U and column B. The difference for each subgame X_1X_2 is calculated based on the assumption that the monopoly would chose the pricing scheme X_i, $i \in \{1, 2\}$ with the highest profit. That is, the total expected equilibrium profit in a UB subgame, for example, is compared to the maximum of the monopoly's U- and B-profits; the total equilibrium profit in a UM subgame is compared to the monopoly's profit under the M-scheme. In the last two columns I report the largest differences across both duopolies for (a) the diagonal subgames UU, BB, and MM(note that these are all the SD-diagonals), and (b) over all possible subgames (note that these are all ND-values).

1% below monopolistic profits. If firms employ pure or mixed bundling, competition can reduce profits by up to 13% if the firms use the same pricing scheme (column "Diagonal" in Table 11) and by up to 21% if the firms use different schemes (column "Overall" in Table 11). This is not surprising given that when firms bundle, a single purchase carries more weight.

Finally, in Table 12 I report market efficiency for all duopoly subgames as well as for the monopoly. Depending on the preference model and pricing scheme, competition increases efficiency by up to 16% (see the last column in Table 12). The distribution of welfare is also affected in a predictable way: under each pricing scheme, competing firms can capture a smaller share of the social welfare compared to a monopoly. As Table 13 shows, unbundling firms lose up to 5% of their welfare share, firms employing only pure bundling or only mixed bundling (BB and MM subgames) lose up to 15%, and the largest difference overall is 22%.

These findings contrast with the case of *ex ante* homogeneous consumers: since pure bundling achieves the first-best solution under both monopoly and duopoly, both market structures are efficient. The firms' profits also fall, however, but for the case of *ex ante* homogeneous consumers, this is a direct consequence of the change in the distribution of the welfare due to competition.[17]

5 Conclusion

In this chapter, I have explored the interaction between competition and the bundling of electronically delivered information goods. One of the main contributions of this work is to focus attention on bundling (monopoly and competitive) with heterogeneous consumers. In particular, consumers vary in the value of their most favored item in a collection as well as in the percentage of items they value positively in that collection. This approach allows me to capture not only the variation in the consumer reservation price for the collection, but also in the amount of correlation among item valuations within the collection, which is an important preference characteristic in the context of (mixed) bundling information goods.

I use an empirical-game methodology developed elsewhere in collaboration with other co-authors to solve the duopoly game. I find that when there are no marginal costs of production for existing information goods, the relative profitability of mixed bundling, pure bundling, and pure unbundling is defined by the preference distribution of the consumers. Generally, the relative performance of these schemes in a duopoly equilibrium – when firms

[17]Fay and MacKie-Mason (1999) show that under duopoly, consumers retain $V_1 + V_2 - V_B$, where V_1 and V_2 are the consumer values for the first and second collections, respectively, and V_B is the value for both collections. Since Fay and MacKie-Mason assume that consumers' valuations of the collections are strictly subadditive, this quantity is positive (as opposed to zero under monopoly). The relative size of the consumer share, however, depends on the shape of the consumer valuation function.

Symmetric Duop.				Non-Sym. Duop.				Mon.		Largest Diff.	
P1-SD				P1-ND				P1-M		Diag.	Overall
	Firm 2(1)				Big Firm						
F. 1(2)	U			Small	U	B	M	Mon.			
U	81	78	80	U	78	79	81	U	77	4	
B	78	85	83	B	78	83	81	B	72	13	
M	80	86	86	M	80	82	84	M	80	6	14
P2-SD				P2-ND				P2-M			
	Firm 2(1)				Big Firm						
F. 1(2)	U	B	M	Small	U	B	M	Mon.			
U	78	77	77	U	76	78	77	U	72	6	
B	77	87	87	B	77	80	80	B	75	11	
M	77	87	87	M	77	80	80	M	75	12	15
P3-SD				P3-ND				P3-M			
	Firm 2(1)				Big Firm						
F. 1(2)	U	B	M	Small	U	B	M	Mon.			
U	78	76	79	U	77	75	77	U	73	6	
B	76	86	80	B	75	79	76	B	70	16	
M	79	80	84	M	78	80	79	M	75	9	16

Table 12: Efficiency (in percent of maximum social welfare). For the duopoly models, a row and a column define a subgame. For example, in the UU subgame (row U and column U) in environment P1-SD, the efficiency is 81%. In the same subgame in P1-ND, it is 78%. In the last two columns, I report (a) the difference between the highest efficiency in duopoly subgames UU, BB, and MM and the monopoly efficiency for the schemes U, B, and M, respectively; and (b) the largest difference over all pricing schemes. For example, the difference for the U-scheme under preferences P1 is $\max(81, 78) - 77 = 4\%$.

Symmetric Duopoly				Non-Symmetric Duop.				Mon.		Largest Diff.	
P1-SD				P1-ND				P1-M		Diag.	Overall
	Firm 2(1)				Big Firm						
F. 1(2)	U	B	M	Small	U	B	M	Mon.			
U	61	54	61	U	64	53	63	U	65	-4	
B	54	48	55	B	57	50	63	B	61	-13	
M	61	55	58	M	61	53	63	M	69	-11	-21
P2-SD				P2-ND				P2-M			
	Firm 2(1)				Big Firm						
F. 1(2)	U	B	M	Small	U	B	M	Mon.			
U	53	59	59	U	55	60	61	U	58	-5	
B	59	51	50	B	56	59	59	B	66	-15	
M	59	50	50	M	56	59	59	M	66	-15	-15
P3-SD				P3-ND				P3-M			
	Firm 2(1)				Big Firm						
F. 1(2)	U	B	M	Small	U	B	M	Mon.			
U	53	48	52	U	54	47	54	U	58	-5	
B	48	36	45	B	52	42	52	B	50	-14	
M	52	45	45	M	52	43	52	M	57	-12	-22

Table 13: Firms' share of actual social welfare (in percent). For the duopoly models, a row and a column define a subgame. For example, in the UU subgame (row U and column U) in environment P1-SD, the total expected equilibrium profits account for 61% of the actual welfare. In the same subgame in P1-ND, it is 64%. In the last two columns I report (a) the difference between the lowest duopoly welfare share in subgames UU, BB, and MM and the monopoly welfare share for the schemes U, B, and M, respectively; and (b) the largest difference over all pricing schemes. For example, the difference for the U-scheme under preferences P1 is $\min(61, 64) - 65 = -4\%$.

compete in price after setting on the scheme – is the same as under monopoly:

- Mixed bundling *strictly* dominates the pure schemes in highly heterogeneous environments, working as a price-discrimination tool that provides incentives for different categories of consumers to self-select into buying individual items or bundles;
- Pure bundling can extract almost as much consumer surplus as mixed bundling through its aggregation effect (effect of reducing buyer diversity) when the number of items consumers value positively (preference breadth) in a collection is constant across the population and only the "intensity" of their preferences varies – that is, when the main source of heterogeneity lies in the reservation price for a collection;
- When the preference breadth varies, pure bundling loses its aggregation power, and pure unbundling can be more profitable, and even as profitable as mixed bundling.

When firms compete, the general mechanisms by which each pricing scheme works to extract consumer surplus remain the same, but price competition can reduce the price-discriminating power of mixed bundling to the extent that pure schemes can result in higher equilibrium profits than mixed bundling. This can happen under a symmetric duopoly when the distribution of consumer preferences is such that a monopolist is almost indifferent between mixed bundling and the pure scheme in question.

Comparing the effect of competition on social welfare and its distribution, I find that it has a negligible impact on the industry profits as well as consumer surplus if items are sold individually. If firms employ bundling schemes – in the pure or mixed form – the effect is noticeable. The market efficiency under duopoly is greater than under monopoly. According to my empirical analysis, it is greater by up to 16% depending on consumer preference distribution and the particular type of bundling scheme. The distribution of social welfare predictably shifts toward consumers under competition, by up to 22%. The largest drop in profits is 21% relative to a monopoly employing the same (combination of) pricing schemes as the duopoly.

APPENDICES

APPENDIX A

Empirical Distribution of Number of Articles Read in a Journal (King and Griffiths, 1995)

Figure A.1 displays a histogram of the empirical distribution of the number of scholarly articles read by a sample of readers (King & Griffiths, 1995) from a collection of about 100 articles. The horizontal axis is the number of articles. The vertical axis is the percentage of the readers who read the corresponding number of articles. The shape of the histogram approximates the density of an exponential distribution.

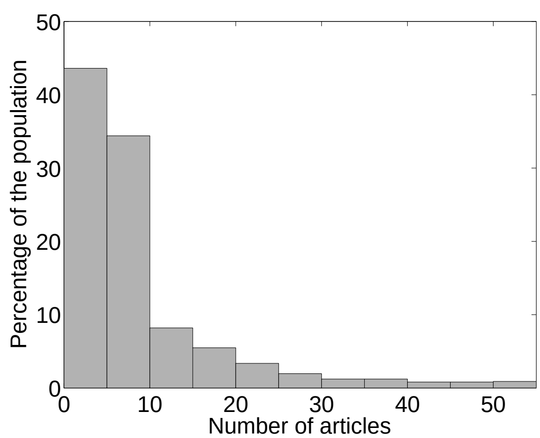

Figure A.1: Distribution of Number of Articles Read in a Journal (King and Griffiths, 1995)

k	Number of articles read	$P(K < k)$
0.05	5	.436
0.1	10	0.78
0.15	15	0.8621
0.2	20	0.9171
0.25	25	0.9508
0.3	30	0.9795
0.4	40	0.9828
0.5	50	0.991
≥ 0.5	≥ 50	1

Table A.1: Data points for Equation (B.2) in Chapter 1. The last column describes the empirical cumulative distribution of the number of scholarly articles read by a sample of readers (King & Griffiths, 1995) from a collection of about 100 articles. *Source: Chuang and Sirbu (1998), p.155.*

APPENDIX B

Parameters of Exponential Preference Distribution Estimated Based on King and Griffith's Empirical Data (1995)

In this appendix, I demonstrate how the parameters for the distribution of k were estimated. The cumulative distribution function of the exponential distribution is given by

$$F(k) = 1 - e^{-\lambda k}, \tag{B.1}$$

where λ is the distribution parameter.

Given γ and assuming $w = 100$ and $N = 100$, we can estimate the mean of the distribution, $\theta = \frac{1}{\lambda}$. Substituting $\theta = \frac{1}{\lambda}$ in (B.1), rearranging the terms and taking logarithm of the both sides of the equation, we obtain

$$k = -\theta \mathrm{Log}(1 - F(k)). \tag{B.2}$$

Let k_i be the preference breadth such that a consumer with access to a collection of a hundred articles would choose to read i of them. Assume also that intensity is constant and equals one hundred ($w = 100$). Then King and Griffiths' data provide eight data points for Equation (B.2) (see Appendix A). The estimated values of k depend on the assumed value of the substitution effect γ. In Table B.1 I report a subset of the estimated values of the mean of the distribution θ, as well as the parameter λ, for a range of values of γ.

γ	$\theta = \frac{1}{\lambda}$	λ
0.	0.0927717	10.7791
0.01	0.0934613	10.6996
0.02	0.0941623	10.62
0.03	0.0948751	10.5402
0.04	0.0956	10.4602
0.05	0.0963374	10.3802
0.1	0.100223	9.97771
0.2	0.109162	9.16066
0.3	0.120132	8.32414
0.4	0.134001	7.46265
0.5	0.152255	6.56794
0.55	0.163819	6.10428
0.6	0.177723	5.62675
0.65	0.194853	5.13208
0.7	0.216652	4.61569
0.75	0.245647	4.07088
0.8	0.286764	3.48719
0.85	0.351227	2.84716
0.9	0.472042	2.11845
0.91	0.510953	1.95713
0.92	0.558992	1.78894
0.93	0.620017	1.61286
0.94	0.700454	1.42765
0.95	0.811852	1.23175

Table B.1: Mean ($\theta = \frac{1}{\lambda}$) and parameter λ of the exponential preference distribution estimated using Equation (B.2) in Chapter 1. The parameter is estimated for a range of γ (column 1) based on empirical data due to King and Griffiths (1995). The collection size is $N = 100$ items. The intensity is assumed to be fixed at $w = 100$. *Data source: Chuang and Sirbu (1998), p. 155.*

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